ISOMORPHIC TESSELLATIONS FOR MUSICAL KEYBOARDS

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ABSTRACT

Many traditional and new musical instruments make use of an isomorphic note layout across a uniform planar tessellation. Recently, a number of hexagonal isomorphic keyboards have become available commercially. Each such keyboard or interface uses a single specific layout for notes, with specific justifications as to why this or that layout is better. This paper is an exploration of all possible note layouts on isomorphic tessellations. We begin with an investigation and proof of isomorphism in the two regular planar tessellations (Square and hexagonal), we describe the history and current practice of isomorphic note layouts from traditional stringed instruments to commercial hex keyboards and virtual keyboards available on tablet computers, and we investigate the complete space of such layouts, evaluating the existing popular layouts and proposing a set of new layouts which are optimized for specific musical tasks.

1. INTRODUCTION

Musical instruments vary in the techniques for providing access to all pitches, but a common method is to provide direct access to each note in a grid or scale. This is the technique for keyboard instruments like the piano, where all available notes are laid out on a linear scale from lowest to highest. While natural and intuitive, this layout has a number of drawbacks: Harmonic relationships between notes are not immediately obvious, and because of the presence of accidentals, scales and chords in each key are played differently meaning that as students learn the piano, they must re-learn scales and chords for each key. Stringed instruments are similar in that for each string all notes are laid out in a linear scale. This is a physical constraint of the instrument, and may in some way have influenced the linear layout of the keyboard instrument. Adjacent strings are related harmonically, which can improve the playability and understanding of harmonic relationships within the scale. Aside from the guitar, which we will consider later, stringed instruments are in fact *isomorphic*, in that harmonic relationships between notes have the same shape regardless of the key in which the note, chord, or scale is played. A perfect fifth is always a single position over or seven positions along a string. This is not the case for the piano, since although all harmonic relationships are the same number of semitones regardless of the key, the shape of these semitones is obfuscated by the location of white and black notes on the keyboard.

Throughout this paper, we will be showing a variety of note layouts in different forms. Since black notes can be spelled more than one way depending on the key and context of the note (*e.g.* C♯ = D♭), and since keyboards do not inherently have a correct spelling for black notes, we will be variously labelling them as sharps, flats, or some combination of the two.

It should also be noted that the musical systems considered in this paper are predominantly western, in that we are not considering microtonality, and we are concentrating on equal-tempered scales.

2. ISOMORPHISM IN MUSICAL INSTRUMENTS

An isomorph is any object that exhibits a uniform shape or structure. Isomorphism applied to musical instruments means that every distinct musical performance is executed in the same way, regardless of key or location. An isomorphic musical keyboard consists of an array of note-controlling elements on which any given sequence and/or combination of musical intervals has the “same shape” on the keyboard wherever it occurs. An example of an isomorphic but non-keyboard musical instrument is a bass guitar. Each string is tuned to an equal interval apart, that of a perfect fourth. With this tuning scheme, all major chords share a common form, as all minor scales are equivalent in structure.

2.1 Uniform tiling of regular polygons

In geometry, a uniform tiling is a tessellation of the plane by regular polygon faces with the restriction that all vertices are identical. Each vertex must be surrounded by the same kinds of face in the same order, and with the same angles between corresponding faces.

There are three regular tessellations that can be formed on a plane. They are built upon three regular polygons: the square, the hexagon, and the equilateral triangle. Of the three tessellations, only the square and hexagon tilings are unidirectional. Equilateral triangle tilings are inherently bidirectional, in that, if you have a triangle that is pointing up, you require a triangle that is also pointing down in order to form a complete tiling. The bi-directionality of triangular tiling violates true isomorphism, as identical chords will appear to have different shapes due to the directionality component of the triangles. It is this feature of triangular tiling that nullifies its use in isomorphic keyboards.

It should be noted here that we are assuming directional homogeneity in these layouts. In other words, if one direction (*e.g.* south) is assigned to a particular interval (*e.g* a descending fifth) then regardless of the starting note, one tile south is always a descending fifth. This constraint makes the layouts easier to learn and guarantees isomorphism in chord-shapes and scale-shapes, in that a player can begin at any note, and a set of tiles in a specific shape will produce the same chord regardless of the starting note. In the future, we may expand the investigation to non-homogeneous layouts, where for example notes in a single direction may alternate or even follow a pattern. It is conceivable that pseudo-isomorphism may be possible with directional non-homogeneity, but this warrants further study.

2.2 Square Tessellations

Most stringed instruments use a square tessellation as the basis of tuning. The horizontal direction is always chromatic, where each fret represents a semi-tone. The vertical direction is often tuned to a perfect fourth or perfect fifth. Figure 1 shows the square tessellation of the violin.

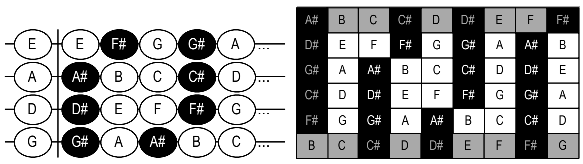

Figure 1. Violin layout and generalized tessellation

There are variations, however. The standard tuning for guitar is not completely isomorphic because the fifth string, B, is tuned one semi-tone less. This is to allow for six-string chords, commonly known as barre chords. The purpose of barre chords is another kind of isomorphism—a player can use a barre chord at any fret and produce the corresponding chord. In this case, a non-isomorphic tuning scheme is adopted to allow for more comfortable, moveable isomorphic chords. The general formula for calculating the musical pattern of squares is based upon the intervals in the vertical and horizontal directions. As stated, it is common that one of the two directions will have intervals of semi-tones. Incorporating the chromatic scale in one of the two directions ensures that all notes are available.

2.3 Hexagon Tessellations

There are comparatively fewer musical instruments that use a hexagonal tiling as opposed to square tiling. The general formula for calculating the musical pattern of hexagons is based upon the intervals in the vertical and horizontal directions. Figure 2 shows the two basic forms of the hexagon tessellation.

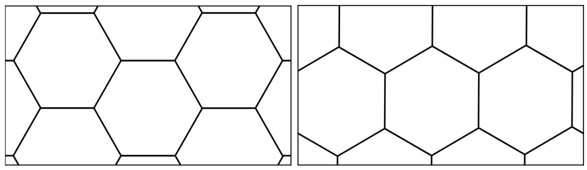

Figure 2. The two forms of hexagon tessellations: vertical (left) and horizontal (right)

In the vertical shape, there is an immediate vertical interval, but no immediate horizontal. In the horizontal shape, the opposite is true. The two differing shapes provide distinct advantages. Like the square tessellations, it is common that either the vertical or horizontal direction will have intervals of semi-tones, although this is not always the case.

3. A GENERALIZED THEORY OF MUSICAL ISOMORPHIC KEYBOARDS

With the two forms of tessellations, we can see that there are constraints on the available intervals in any given direction. This section will show that in both square and hexagonal layouts, all possible layouts can be represented by a horizontal interval and a vertical interval.

3.1 Square Layouts

As noted earlier, intervals in one direction are constrained to be equal, so that the isomorphism is guaranteed. This also means that we constrain intervals in one direction to be the opposite of intervals in the other direction. Upward intervals are the opposite of downward intervals, and left and right intervals are also constrained to be opposites. If we use $+V$ to indicate the upward or vertical interval, in semitones, and $+H$ to indicate the right or horizontal interval, we can see from Fig. 3 that these two intervals fully define the notes in a square tessellation. Starting at a note U, any closed loop of note transitions will come back to the original note.

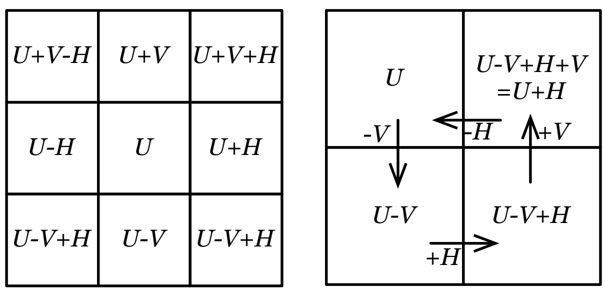

Figure 3. Generalized form of the square isomorphic tessellation. Any closed loop path will return to the original note.

3.2 Hexagonal Layouts

The hexagonal layout can similarly be represented by a vertical and a horizontal interval, although to show this we must do some analysis. Figure 4 shows the orientation of the horizontal and vertical intervals. We also have another interval, A, which interrupts the line between H and V. All intervals are relative to U.

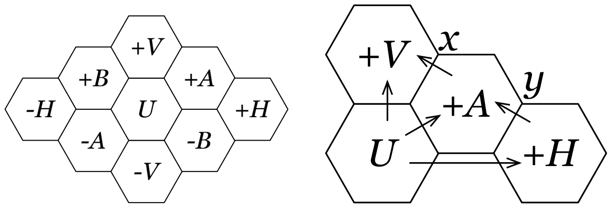

Figure 4. The triangle of interest in the vertical layout

For our layout to work, any closed circuit must bring us back to the original layout. If we consider the triangle of interest shown in Fig. 4, there are two triangles which include the still unknown interval A (assuming we have defined V and H). These triangular circuits are: $U+A+x = U+V$ and $U+H+y = U+A$. We can simplify these equations in two ways, First, by removing U from each side we are dealing with the intervals themselves, unrelated to any specific root note. This further emphasizes the isomorphic nature of these layouts. The second simplification is to note that if intervals in the same direction must be the same, then $x = y$. Therefore $A + x = V$ and $H + x = A$. If we solve first for x and substitute, we see that $H + (V - A) = A$, and further that $A = (H + V)/2$. This proves that all notes on a hexagonal grid can be defined purely by the horizontal and vertical intervals as indicated in Fig. 4. Further, it shows that if we have two notes (H, V) in a line separated by a single note A, the interval between H and V must be an even number of semitones. The same proof can be done for the B interval shown in Fig. 4, as well as with the inverse hexagonal layout, although in this case H would share a vertex with U, and V would be two notes away.

These generalizations will be used to analyze a number of historical examples of isomorphic musical interfaces, before exploring the complete space and performing a theoretical analysis of which particular layouts may be better for which musical purposes.

4. HISTORY OF ISOMORPHIC MUSICAL KEYBOARDS

Although stringed instruments can be considered to use a square tessellation layout, they are not usually considered a musical keyboard as such. There are few square-tessellation musical keyboards available. You can commonly find midi-controllers that form a square lattice but these are usually used for drum machine, music sequencing, or other rhythmic purposes. There is no standard in square grid musical keyboards and it is still generally underdeveloped. Hexagon keyboards seem to have caught on a lot faster than square keyboards, as there are several variations of them on the market today.

Hexagonal isomorphic keyboards in a musical instrument probably stem from the early 19th century accordion or concertina. The accordion utilized an offset grid of circular buttons which were organized on a hexagon lattice. In the later 19th century, Paul von Jankó reinvented the musical layout of a piano[1], by creating a multi-row keyed instrument (Fig. 5), which consists of alternating rows of black and white keys.

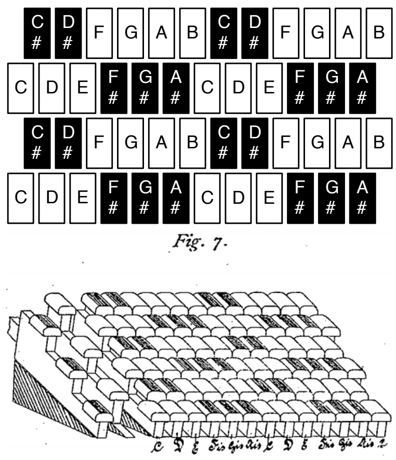

Figure 5. Jankó layout and original patent figure

It was the first isomorphic piano-like keyboard. This layout is commonly known as the Jankó layout, and can be defined as H=2, V=0. Today, there are several commercially available hexagon keyboards: the C-ThruAXiS-49, the Opal Chameleon, and the ThumMusic System "Thummer" [2] . The AXiS-49 and Opal Chameleon keyboards use the Harmonic Table note layout (V=7, H=1). Although the Thummer is not currently in production, it is filed under several patents and uses the Wicki-Hayden layout (V=12, H=2).

Figure 6. The thummer and the axis49, commercially available isomorphic hexagonal keyboards

The Harmonic Table note layout has been well known since the 18th century. Leonhard Euler had developed a tone matrix which he called the Tonnetz [3] and it is found to be equivalent to the harmonic table. The harmonic table integrates a perfect fifth interval in the vertical direction with a semi-tone interval in the horizontal direction. The harmonic table is below:

The Wicki-Hayden layout is the arrangement that is commonly used on accordions and concertinas, shown in Fig. 7. It was first conceived by Kaspar Wicki and a variant was patented by Hayden in 1896 [4]. The Wicki-Hayden layout integrates octaves in the vertical direction, and whole-tone intervals in the horizontal direction. It uses the "inverse" hexagon arrangement.

Interestingly, the standard computer keyboard has also been used to experiment with hexagonal layouts. Because

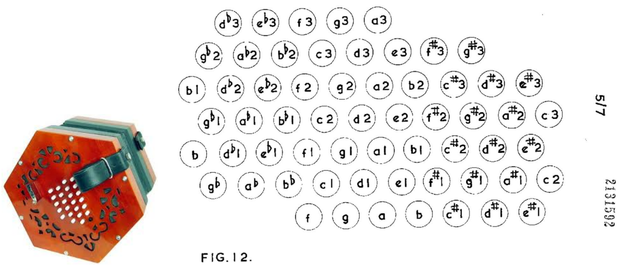

Figure 7. A concertina, and Hayden's original patent figure for the Wicki-Hayden layout

each row of keys is offset by about half a key from the previous row, the arrangement is the same as the horizontal hex layout and it is possible to simulate, for example, the Wicki-Hayden layout by assigning each key to a separate note (F=$b^\flat$; G= c; H=d, T=f; Y=g etc.).

Apart from hardware adaptations of hexagonal music systems, there exists software for Apple products which provide the same layout. Michael Eskin has developed the mJammer, iJammer, and HexJam applications, all of which utilize the Wicki-Hayden layout [5].

Musix, developed by Shiverware interactive (launched by two of the authors of this paper), is a fully-customizable multiple-layout isomorphic keyboard instrument. With the Musix software, over a thousand unique hexagonal isomorphic layouts can be created. Several novel isomorphic schemes have been discovered through the use of the Musix software, and testing of the usability of these layouts is planned for future work.

5. MELODIC AND HARMONIC ANALYSIS

The square and hexagon isomorphic grids will be analyzed in terms of: (1) melodic ability and (2) harmonic ability.

5.1 Melodic Ability

A melodic line is a succession of notes forming a distinctive sequence. This sequence can form either a scale or a tune (a melody). In either case, it is a musical phrase that develops horizontally, one note at a time. In music theory, there are two basic progressions that are used to characterize scales–the diatonic and the chromatic.

The diatonic scale is a seven note octave-repeating musical scale comprising five whole steps and two half steps for each octave. The "white-keys" on a piano form a diatonic scale, and can be used to construct the major scale and its seven derived modes, including the natural minor scale. The diatonic scale forms the foundation of music as we know it today. The chromatic scale is composed of twelve equally spaced pitches, each a semi-tone apart. It contains all of the notes within one octave. It has no root, or tonic, due to the symmetry of its equally spaced tones. The chromatic scale contains the twelve tones that are repeated every octave.

Most melodies are composed of mostly diatonic scales with chromatic or "accidental" inflections. By observing the ease of playing both diatonic and chromatic scales, one can gain a sense of the level of melodic ability supported by each isomorphic layout.

5.2 Harmonic Ability

If melody is said to be the horizontal aspect of music, then harmony is the vertical aspect. Harmony is the instantaneous use of two or more simultaneous tones. All chords are in fact a musical harmony. There are infinite amounts of harmonies that can be achieved through any number of discrete tones. In music theory, there are several harmonic combinations that are found in the music of all ages. These are the octave, the dominant triad, and the dominant seventh.

The octave is an interval between two musical notes where one of which has twice the pitch of the other. They are perceived to be the same note. It is usually considered the most consonant harmony, besides perfect unison. The dominant triad is a chord composed of the root note, the diatonic third and the perfect fifth above it. The third can be either major or minor, depending on the leading tone of the scale. The root and perfect fifth do not vary. There are three versions of the triad. Using C major as an example, we have the dominant root position of C–E–G, the first inversion E–G–C, and the second inversion G–C–E. The dominant triad is the most commonly found chord in music. The dominant seventh is a chord composed of the root note, major third, perfect fifth, and minor seventh. The dominant seventh is found almost as often as the dominant triad. It is also one of the most fundamental four-note chords. By observing the ease of playing the octave, the major and minor dominant triads, and the dominant seventh chord, one can gain a sense of the harmonic ability provided by each isomorphic layout.

6. SQUARE LAYOUTS

Examples of the square layouts are shown in Fig. 8. Although there are many possible layouts, some are *degenerate*, meaning not every note is present. We restrict these layouts to only non-degenerate cases.

6.1 Semi-tone, Semi-tone (V: +1 H: +1)

Melodic: For the experimental composer who wants complete control of the chromatic scale. The diatonic scale can be played on the positive diagonal slope or along one semitone axis. The dual-linearity makes it easy to play scales in both the vertical and horizontal axis. Notes are repeated on the negative diagonal slope. There is no inverse for this tessellation.

Harmonic: Close harmonies, such as triads, are in range when played diagonally. Triads may be difficult to play on one axis due to the spacing of the notes. The dominant seventh is even more difficult to play due to note spacing. Discordant harmonies are abounding with close clustering. The additional octave may also be difficult to play with one hand, depending on the size of the squares.

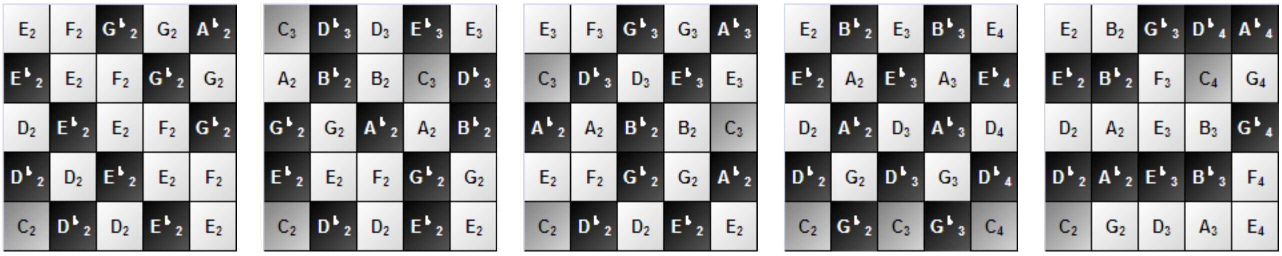

Figure 8. Examples of square layouts: +1+1; +3+1; +4+1; +1+6; +1+7;

6.2 Whole-tone, Semi-tone (V: +2 H: +1)

Melodic: The accidentals are grouped together on the whole-tone axis, which makes the diatonic scale easy to locate and perform. On the inversed matrix, the layout partially mimics a piano as the Db is located above and to the left of the D. This is useful for the composer who wishes to work with purely whole-tone and semi-tone scales. The chromatic scale is easy to play in the original layout, as a horizontal approach is taken to play the scales. Conversely, the diatonic scale is easier to play in the inverse layout as it is more compact but yet still played horizontally.

Harmonic: On both layouts, chords requiring the octave need to be performed diagonally. Triads are easily located but there seems to be no ideal fingering. The dominant seventh is not difficult to play. The inversed layout may provide slightly better fingerings for chords as the slope is less inclined than the original arrangement.

6.3 Minor Third, Semi-tone (V: +3 H: +1)

Melodic: The diatonic scale can be played a number of ways in both the original and inversed layout. There seems to be no best approach to play the scale. The chromatic scale is easily accomplished by playing three notes in the semi-tone axis, then moving up to the minor third interval. Scales are easier to play in the original layout because a horizontal approach is taken.

Harmonic: The inversed arrangement is the ideal square layout for harmonic combination. Major and minor triads are immediately at hand. The octave is readily found on the minor third axis, and the pattern repeats every four minor third intervals. The dominant seventh chord is easy to play. All harmonic combinations lend themselves well with to the inversed layout, because it assumes a horizontal approach will be taken to playing the chords. In the original layout, chords are more difficult to play because the hand is forced to play the chords in the vertical direction, sideways.

6.4 Major Third, Semi-tone (V: +4 H: +1)

Melodic: The diatonic scale can be completed in three three-note movements. The chromatic scale is played four-notes at a time, before moving up the major third interval. Both scales are easier to play in the original layout.

Harmonic: The location of the perfect fifth and diatonic thirds from the root makes this layout less ideal for harmonic combination. Triads are in reach but there appears to be no ideal fingering for them. The octave is found on the major third axis, and the pattern repeats every three major third intervals. The dominant seventh chord requires more difficult fingering.

6.5 Perfect Fourth, Semi-tone (V: +5 H: +1)

Melodic: The original layout is ideal for playing and constructing melody. Nearly every scale contains the perfect fourth, so this layout lends itself tremendously well for the playing of scales. This arrangement happens to be the same tuning layout used by many stringed instruments. The diatonic scale is extremely easy to play. Three notes are played for every perfect fourth interval. The chromatic scale can be played by playing five notes on the semi-tone axis, then moving up to the perfect fourth interval. The inversed layout is not as good for playing scales or melodies because it requires one to play on the vertical axis.

Harmonic: On the original layout, most combinations are quite easily played. The diatonic third is played on the semi-tone axis and the perfect fifth is played on the interval above. Directly above the perfect fifth is the octave, which makes the octave easy to add. The drawback is that the dominant seventh chord is challenging to play. In general, chords are more difficult to play on the inversed layout.

6.6 Tri-tone, Semi-tone (V: +6 H: +1)

Melodic: All scales are easier to play in the original layout because they are played horizontally. The diatonic scale can be completed in just two movements in the vertical direction. Alternatively, the diatonic can be played by three notes on the semi-tone axis, then moving up the diatonic interval and back one semi-tone. The chromatic scale requires six notes to be played on the horizontal before moving up the tri-tone interval. This implies that one finger may have to play two notes. Alternatively, five notes can be played and the pattern is moved back on semi-tone on the tri-tone interval above. This layout forms an interesting checkerboard pattern which may help to provide a new approach to melodic structures.

Harmonic: The original layout is better for constructing chords, as they can be played horizontally. Major and minor triads are easy to play when the perfect fifth is played on the interval above. The dominant seventh isnt difficult to play. However, the location of the octave makes it quite difficult to add an octave to any chord structure.

6.7 Perfect Fifth, Semi-tone (V: +7 H: +1)

Melodic: The original layout is better for the performance of melodies because it is done horizontally. The diatonic scale can be completed in two or three vertical intervals. Scales require a significant jump (5 tiles back and one tile up) to get from one row to the next, but historically this has not been a problem for violin, viola, cello and mandolin players, possibly because the physical distance from one "tile" to the next is small on these instruments. This may be why the double bass uses +5+1 instead of +7+1.

Harmonic: Once again, the original layout is better for harmonic combination because chords are played horizontally. The perfect fifth is considered the most consonant musical harmony besides unison and the octave, and for that reason it is commonly found in most chords. With this layout, the perfect fifth is located immediately above the selected note. If the root note and perfect fifth are played with one finger, triads and even the dominant seventh are quite easy to play. The location of the octave makes it difficult to include, however.

6.8 Octave, Semi-tone (V: +12 H: +1)

Melodic: This arrangement forms a completely chromatic interface. The diatonic scale and chromatic scale are to be completed on the semi-tone axis. The original layout is better for melodic ability as the scales are performed horizontally.

Harmonic: On both layouts, chords may be somewhat difficult to play due to the spacing between octaves. However, interesting chord voicings that span several octaves can be quite easily created.

7. HEXAGON LAYOUTS

Each hexagonal layout presented here uses a specific orientation of the hexagonal tessellation, as well as a specific vertical and horizontal interval. We also show the inverse of each layout, which corresponds to the same vertical and horizontal layout in the opposite hexagonal tessellation. We beg the reader's indulgence in our naming of the three new proposed layouts, each was build separately by a contributor to the project, with specific musical intentions in mind. As with square layouts, we restrict the presentation here to "interesting", non-degenerate cases.

Each layout is evaluated in terms of both Melodic and Harmonic features. As an example, a collection of triads in the Harmonic layout are presented in Fig. 9 and the major scale for the Harmonic layout is presented in Fig. 10.

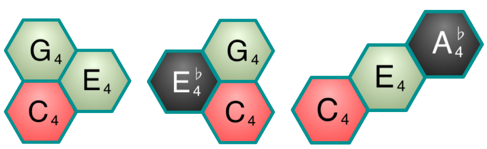

Figure 9. Triads for the Harmonic layout (see Fig. 12): Major, Minor, Augmented

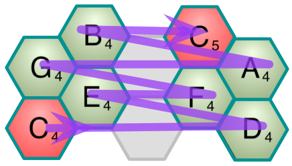

Figure 10. A major scale in the Harmonic layout

7.1 Wicki–Hayden (V: +12 H: +2)

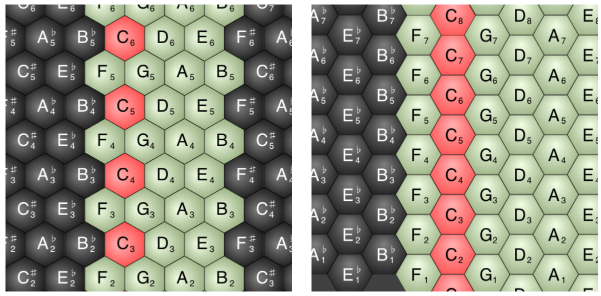

Figure 11. Wicki-Hayen (horizontal) layout: original, inverse

Melodic: The diatonic scale is intuitive to play on the original Wicki-Hayden layout. The white keys make up the center of the keyboard and the accidentals are on either side. The major scale takes three vertical movements to complete. The scales are played horizontally. Most intervals are whole steps which may make mistakes sound more consonant. The chromatic scale requires a more awkward movement, implying that non-diatonic scales and melodies containing accidentals will be a challenge to perform. On the inversed layout, both the diatonic and chromatic scales are more difficult to play, due to increased spacing between notes.

Harmonic: On the original, major and minor triads are easy to play, which is made possible by the immediate interval to the perfect fifth from the root. The fingering recommended for the triads is to play the root with the middle finger, the fifth with the ring finger, minor third with the pointer finger, and major third with the pinky. However, the location of the octave makes it difficult to add on to an existing chord. On the inversed layout, all of the chords are more difficult to play, due to increased spacing between notes. Only the original layout is the true Wicki-Hayden layout.

7.2 Harmonic Table (V: +7 H: +1)

Melodic: On the original, the diatonic scale is played in two-note movements on the horizontal axis, moving up the vertical as the scale ascends. Looking at the C in the bottom left corner, it is apparent that the major scale is played with high symmetry. The same is true for the inversed layout, although it is more condensed. Semi-tones are found in the horizontal axis, which makes the chromatic scale easy to play.

Harmonic: The original tessellation has interesting harmonic combinations. One may quickly notice that the ma-

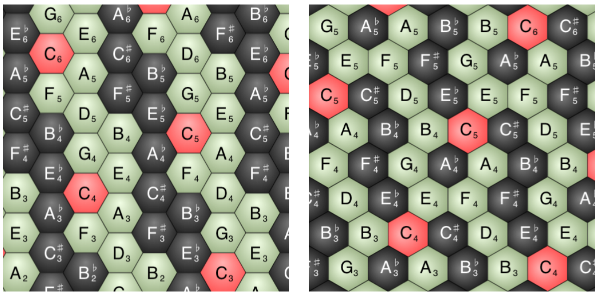

Figure 12. Harmonic layout (vertical) : original, inverse

jor and minor triads are formed by making small triangular clusters. This arrangement allows the playing of chords with only one finger. The dominant seventh chord is also easy to play. The octave spacing is close enough that one should be able to add it without much effort. This layout works extremely well for "jamming" as all the immediate surrounding notes are consonant. On the inversed tessellation, the harmonic combinations are not as compact–but this is not necessarily a bad thing. The triads can no longer be played with just one finger, but if one plays the perfect fifth that is a bit further to the right of the root, the chords are fairly comfortable to play. With this fingering, adding the octave to the chord is easily done. Performing the dominant seventh chord is more difficult.

7.3 Gerhard (V: +1 H: +7)

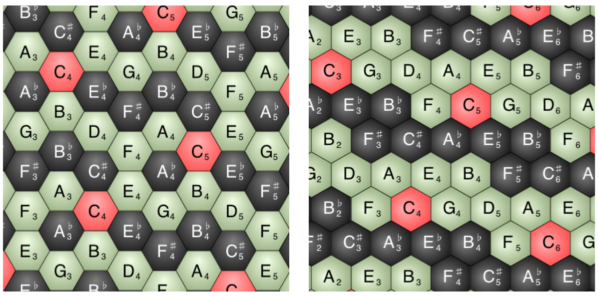

Figure 13. Gerhard layout (vertical): original, inverse

Melodic: This arrangement is similar to the Harmonic layout, its main difference being that semi-tone intervals take up the vertical axis where we had perfect fifths previously. The diatonic scale is easily completed by playing two notes in the vertical axis then moving to the right as the scale ascends. The major scale is again seen to be symmetrical. On the original tessellation, the scale is more compact. The downside is that there is no easy way to play the scales horizontally.

Harmonic: The original arrangement lends itself extremely well to major and minor triads. The major and minor thirds are adjacent and are located in between the root and perfect fifth. The dominant seventh is also in the vicinity. With this arrangement, adding the octave to the chord may be challenging. In the inverse arrangement, the triad chord shapes are triangular, as in the harmonic layout, and able to be played with one finger. The dominant seventh chord is also easy to play, but adding the octave to the chord is difficult.

7.4 Park (V: +1 H: +5)

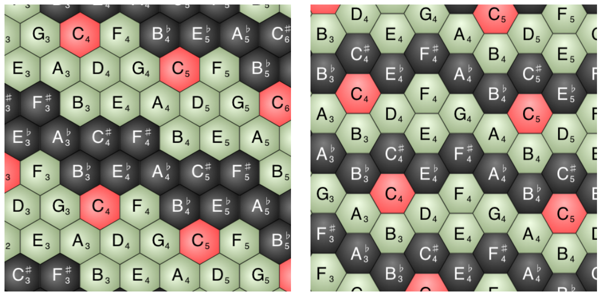

Figure 14. Park layout (horizontal) : original, inverse

Melodic: The diatonic scale is played on the negative slope. Semi-tones are in the vertical direction, which makes the chromatic scale easy to play. The inverse layout is superior because the diatonic scale is less sloped and more linear, and the chromatic notes are located directly above the root.

Harmonic: Major and minor triads are easy to play although their chord shapes are dissimilar. The dominant seventh is difficult to play. The location of the octave is convenient in that it can be added on without too much strain. In the original layout, the chords are more compact.

7.5 Maupin (V: +1 H: +3)

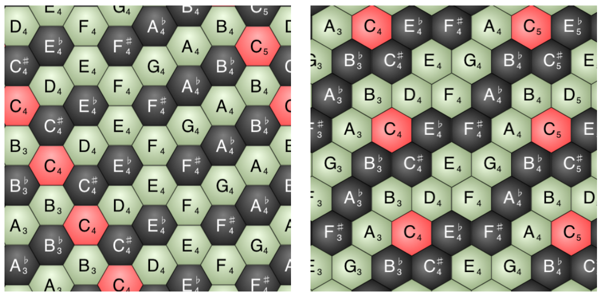

Figure 15. Maupin layout (vertical) : original, inverse

Melodic: The diatonic scale is played on the positive slope. Semi-tones are in the vertical direction, which makes the chromatic scale easy to play. In the inverse arrangement, it takes on a piano-like form. The inverse arrangement is superior because the diatonic scale is less sloped and more linear, and the semitones are located directly above the root.

Harmonic: In both arrangements, the triads require more of a stretch as compared to other layouts but may actually be more comfortable to play. The major third suggested is the one located on the diatonic scale, with the perfect fifth located beside it. The advantage is that the minor thirds are adjacent to the major thirds, which makes changing between major and minor chords easy. The added octave and dominant seventh chord are also surprisingly easy to execute in both layouts. The inverse arrangement offers chord shapes that are more comfortable to play.

8. ANALYSIS, CONCLUSIONS, AND FUTURE WORK

Of the square tessellation, we easily conclude that intervals of perfect fourths in the vertical axis and semi-tones in the horizontal axis provides the most advantageous approach to constructing and performing melody. There are several reasons for this, apart from its widespread use in stringed instruments. The diatonic scale forms a repeatable pattern when played three notes per every vertical jump.This layout also provides the ideal arrangement for the chromatic scale, as five notes per interval jump are required. This means the chromatic scale can be performed with one hand, quickly and easily, with no horizontal movement. Of the hexagonal tessellation, the harmonic table layout and the Maupin layout provide distinct advantages for melodic performance. The harmonic table layout excels in that the diatonic scale can be performed as two notes per every vertical jump. It requires little horizontal movement, and the pattern is highly symmetrical. The main advantage of the Maupin layout is that it appears to have the same layout found on a regular piano keyboard, albeit on a positive slope. The diatonic scale is easy to pick out although it requires more of a horizontal movement. The chromatic scale is easier to perform than the harmonic table layout because semi-tones are found immediately adjacent to the root note. This implies that accidental inflections would be significantly easier to perform.

Of the square tessellation, the inverted arrangement of semi-tones in the vertical direction, and minor third intervals in the horizontal direction provides the best layout for harmonic combination. This layout is the second layout presented in Fig. 8. The octave is located on the same axis as the root, which makes it easy to add to any harmonic combination. In between the octaves, the perfect fifth is found on the interval above. This is an ideal location of the perfect fifth, as the major thirds is found directly to the left, and the minor seventh to the right. This is the most comfortable way to play a dominant seventh chord on a square tessellation. In fact, all chords are found to be extremely comfortable to play with this tiling.

With the harmonic table layout, triads can be performed with one finger. It is most suited for “jammers”, a style of keyboard which ensure consonant notes are surrounding the root note. The dominant seventh is easy to play, and octave spacing is close. The downside to the harmonic layout is that harmonically complex chords are difficult to perform as all of the notes are clustered close together. The original Gerhard layout provides triads that are easy to execute, as the minor and major thirds are directly adjacent to each other. The dominant seventh chord and added octave are also easy to perform with this layout. However, as found with the harmonic table layout, the close clustering of these chords makes the hand bunch up when more harmonically complex chords are tried. With the Maupin layout, chords are much more spread out and are played similarly to that found on a piano. The minor and major thirds are found immediately adjacent which simplifies the playing of triads. It takes on the same ”comfortable” chord shapes that are found in the square tessellation of minor third intervals. The dominant seventh with added octave is found to be easily performed. The advantage of this layout is that the chordal note spacings are more spread out, and therefore feel more relaxed in technique.

We recognize that the analyses of the various layouts are at this point theoretical, so we plan to explore how individual players learn, play, and compose with these layouts. We will do controlled subject trials with both musicians and non-musicians, studying perceptual, musical, and bio-ergonomical considerations. Additionally, as discussed, we plan to study the potential pseudo-isomorphisms that may be present in any triangular or otherwise directionally non-homogenous layouts.

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