# A COMPUTATIONAL MODEL THAT GENERALISES SCHOENBERG'S GUIDELINES FOR FAVOURABLE CHORD PROGRESSIONS

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## ABSTRACT

This paper presents a formal model of Schoenberg's guidelines for convincing chord root progressions. This model has been implemented as part of a system that models a considerable part of Schoenberg's Theory of Harmony. This system implements Schoenberg's theory in a modular way: besides generating four-voice homophonic chord progressions, it can also be used for creating other textures that depend on harmony (e.g., polyphony).

The proposed model generalises Schoenberg's guidelines in order to make them applicable for more use cases. Instead of modelling his rules directly (as constraints on scale degree intervals between chord roots), we actually model his explanation of these rules (as constraints between chord pitch class sets and roots, e.g., whether the root pitch class of some chord is an element in the pitch class set of another chord). As a result, this model can not only be used for progressions of diatonic triads, but in addition also for chords with a large number of tones, and in particular also for microtonal music beyond 12-tone equal temperament and beyond 5-limit harmony.

## 1 INTRODUCTION

Computational models of music theory are interesting for at least two reasons. Firstly, declarative models improve our understanding of the theory. Secondly, computational models can also be used as tools in the composition process.

Tonal harmony has often been modelled declaratively. Surveys on this subject are provided in [7] and [2]. Particular important is the system CHORAL [3, 4], which creates four-part harmonisations in the style of Johann Sebastian Bach for given choral melodies. It implements about 350 rules, and received much attention for the musical quality of its output. The music representation MusES [6] has been used for harmonic analysis, melody harmonisation, and modelling jazz improvisation. A number of other systems also do automatic melody harmonisation. For example, [13] proposes a lucid system with a small set of 20 rules, which creates four-part harmonisations of a choral melody. [10] describes another system that automatically harmonises a given melody. Coppelia [14] creates homophonic chord progressions, which additionally feature a rhythmical structure. [9] presents a further system that generates choral harmonisations in the style of Johann Sebastian Bach.

The authors of [7] claim that the "technical problem of four-voice harmonization may now be considered as solved". However, existing systems only solve a special subtask of harmony: instead of creating a harmonic progression from scratch, these systems harmonise a given melody, most often creating a new chord for each melody note (choral harmonisation). Also, most existing systems create solutions that are very modest musically. For example, only the systems of Ebcioglu and Phon-Amnuaisuk address modulation at all. Even Ebcioglu's highly complex system CHORAL formalises possible chord progressions simply by quasi a transition table that only allows for common progressions. For example, in major the degree $II$ is mostly followed by $V$, and by $I$ or $VI$ only under specific conditions, [1] while the diminished triad $vii^o$ is always followed by the tonic $I$ [3, p. 240 f]. Yet, these chords can progress to any degree in principle.

We argue that modelling harmony is not a solved problem yet. Harmony is a highly complex phenomenon as demonstrated by the library of harmony textbooks available. Still missing is a system that models

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[1] Ebcioglu's rule set states that $II$ can only be followed by $I$ or $VI$ if some non-bass voice moves by a third skip from the $VI$ (the fifths of the $II$ chord) to the $I$.

harmony on the level of abstraction presented by acclaimed theory texts like [11]. For example, using a transition table for chord progressions is a useful shortcut, but theorists like Schoenberg teach us better alternatives.

This paper describes a system that models a considerable part of Schoenberg's Theory of Harmony [11]. This system generates self-contained harmonic progressions – instead of harmonising existing melodies – and can so create the harmonic backbone of new compositions. The harmonic model is modular for applications beyond four-part harmonisations. It can serve as a foundation for modelling musical styles that depend on harmony (e.g., Baroque counterpoint), and can also be an interesting composition tool. Schoenberg has been selected as theoretical foundation, because this textbook is unique in its focus on writing convincing chord progressions, instead of focusing on analysis, melody accompaniment or figured bass (as many other harmony textbooks do).

For space limitation, this paper details only one aspect of Schoenberg's theory, but this aspect is of particular importance. In the chapter "Some Directions on Writing Favourable Progressions" ("Einige Anweisungen zur Erzielung günstiger Folgen" [11, p. 134 ff]), Schoenberg presents guidelines on root progressions that result in particularly convincing chord sequences. [2] This paper formalises these guidelines. To our knowledge, these guidelines have never been modelled before. The model has been implemented in Strasheela [1].

In addition, this model generalises these guidelines for chords with a large number of notes (as long as we know their root), and in particular also for microtonal music beyond 12-tone equal temperament and beyond 5-limit harmony [8]. Schoenberg discusses his guidelines only in the context of triads in a diatonic scale as he formulates his rules on the scale degree of chords. Nevertheless, his detailed explanation of these rules are more general. Instead of formalising Schoenberg's rules directly, this paper actually models his explanation of these rules. Doing so makes his concepts applicable for more music, but also puts some corner cases of classical harmony in a new light.

## Plan of Paper

The rest of this paper is organised as follows. Schoenberg's guidelines on root progressions are recapitulated in Section 2. Section 3 presents a model that formalises and generalises these guidelines. Musical results are presented in Section 4. The paper ends in a summary (Section 5).

[2] A summary of these guidelines can also be found at the beginning of his book "Structural Functions of Harmony" [12].

## 2 THE MUSIC THEORY

Schoenberg distinguishes three root progression cases: ascending, descending and super-strong progressions. In an *ascending* progression, the chord root progresses a fourth up / a fifths down (e.g., $V-I$) or a third down (e.g., $I-VI$). Schoenberg calls such progressions also *strong* and advocates their unreserved use.

A *descending* progression – quasi a reversed ascending progression – proceeds a fifths up ($I-V$), or a third up (e.g., $I-III$). Schoenberg avoids the term *weak*, but nevertheless discourages their unconfined use. Instead, Schoenberg recommends that in a sequence of three chords $C_1, C_2, C_3$ the sequence $C_1, C_2$ can only be descending if $C_1, C_3$ is ascending (e.g., $III-V-I$). In that case, the purpose of the middle chord $C_2$ is similar to the purpose of a passing note in a melody.

Finally, a *super-strong* progression connects two chords whose root are a second apart (e.g., $V, VI$ or $V, IV$). Such progressions are typically used in a deceptive cadence. Because their quality can be considered too strong, Schoenberg advises to use them sparely.

Schoenberg argues at length possible reasons for the different qualities of these progressions. These will be briefly reported below when they are formalised.

## 3 THE FORMAL MODEL

This section presents the formal model of Schoenberg's guidelines for favourable chord progressions. The model implements Schoenberg's explanation instead of his actual rules.

Our full system defines a rich and highly extendable music representation designed for modelling a wide range of music theories. This representation provides a rich collection of score objects including elements such as notes, or rests, analytical concepts such as intervals, scales, chords, or meter, grouping concepts such as containers that arrange their content sequentially or parallel in time, as well as concepts for organising musical form such as motifs. For brevity, this section introduces only a small fraction of this representation that is sufficient for modelling Schoenberg's guidelines on root progressions.

A chord $C$ is a score object that represents the analytical notion of a chord or harmony. This analytical object is silent when the score is played, but influences the pitches of note objects. For the present model, a chord object encapsulates only two attributes: the pitch classes of the chord *pcs* and its root *root*. Both these attributes are variables in the logic or constraint programming sense. *root* is a finite domain integer, and *pcs* is a finite set of integers. In Schoenberg's theory, the root of a chord is always a member of its pitch class set: $root(C) \in pcs(C)$. For example, the root of the diminished triad $\{B, D, F\}$ is $B$.

We will now model Schoenberg's notion of ascending, descending and super-strong progressions as Boolean functions on pairs of consecutive chord objects $C_1$ and $C_2$. Schoenberg explains that in an ascending progression the root of a former chord is "over-ruled" by a new root in the following chord. Formally, the root of $C_1$ is also a member of the pitch class set of $C_2$, but the root of $C_2$ was not contained in $C_1$ (Figure 1).

$$\begin{aligned} isAscending_1(C_1, C_2) := \quad & root(C_2) \notin pcs(C_1) \\ & \wedge root(C_1) \in pcs(C_2) \end{aligned}$$

**Figure 1**. Ascending progression: the root of the first chord is also contained in the second chord, but the root of the second chord is new

In a descending progression, the root of the second chord is a "parvenu" according to Schoenberg, the ruler (root) of the first chord quasi backs down to one of his former "subjects". Formally, a non-root pitch class of the first root becomes root in the second chord (Figure 2).

$$\begin{aligned} isDescending(C_1, C_2) := \quad & root(C_2) \in pcs(C_1) \\ & \wedge root(C_1) \neq root(C_2) \end{aligned}$$

**Figure 2**. Descending progression: a non-root pitch class of the first root becomes root in the second chord

In ascending and descending progressions, chords share common pitch classes (what Schoenberg calls a "harmonic band"). In a super-strong progression, all pitch classes of the second chord are new and there are no common pitch classes (Figure 3).

$$isSuperstrong(C_1, C_2) := \; pcs(C_1) \cap pcs(C_2) = \varnothing$$

**Figure 3**. Super-strong progression: two consecutive chords do not share any pitch classes

Schoenberg only discusses these three cases, because he discusses only diatonic triads. However, there exist two further cases in principle. Firstly, two different chords can share the same root as in $C - Cmin$ (Figure 4).

Secondly, outside the set of diatonic triads there exist progressions that are connected by a harmonic band,

$$isConstant(C_1, C_2) := \; root(C_1) = root(C_2)$$

**Figure 4**. Constant progression: two (possibly different) chords share the same root

but that are neither ascending nor descending progressions according to the definitions above. For example, the triadic progression $C - E\flat$ shares common pitch classes (the tone $G$), but it belongs to none of the categories above. In our subjective assessment these progressions also feel strong, like the ascending progressions. Instead of introducing a fifths category, we therefore propose a generalised version of *isAscending* as an alternative that includes also those progressions where the second chord has a new root, but the root of the first chord is not contained in the second (Figure 5).

$$\begin{aligned} isAscending_2(C_1, C_2) := \quad & root(C_2) \notin pcs(C_1) \\ & \wedge pcs(C_1) \cap pcs(C_2) \neq \varnothing \end{aligned}$$

**Figure 5**. Ascending progression (generalised version): the root of the second chord is new, but both chords share common pitch classes

Following some speculation in Schoenberg's treatise, we also implemented a progression strength measurement that combines all the cases above in a single numeric measurement, and that for a more fine-grained discrimination additionally takes the cardinality of the harmonic band into account, weighted against the total number of chord pitch classes.

Finally, Figure 6 implements Schoenberg's recommendation that a descending progression is resolved as quasi a "passing chord". [3]

$$\begin{aligned} & resolveDescending(C_1, C_2, C_3) := \\ & \quad isDescending(C_1, C_2) \Rightarrow isAscending(C_1, C_3) \end{aligned}$$

**Figure 6**. Resolve descending progressions quasi as "passing chords"

These functions have been implemented as constraints in our system: they can be combined with other constraints and a solver can find one or more solutions. For efficiency, our constraint programming system uses

[3] Schoenberg recommends this strict version of the rule, a relaxed version also permits interchange progressions (e.g., $I - V - I$)

constraint propagation and dynamic variable orderings customised for this problem [1] (e.g., the solver progresses from "left to right" in score time but for simultaneous score objects always first determines rhythmic parameters, then scale or chord parameters and finally the actual note pitch classes and octaves).

## 4 RESULTS

This section provides musical results that have been generated by a system that implements the presented model. Figure 7 shows a chord progression that was generated with the proposed model. Whereas we only formalised Schoenberg's root progression guidelines in this paper, generating this example obviously required modelling further aspects of Schoenberg's theory such as part leading rules (e.g., avoid parallels, and keep the harmonic band in the same voice and octave), or the treatment of chord inversions.

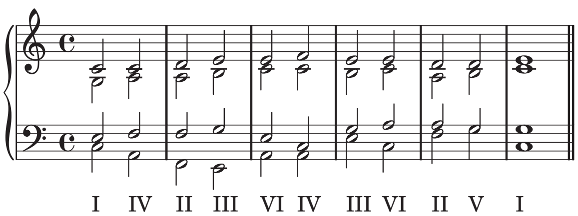


**Figure 7**. Chord progression generated by the presented model

Nevertheless, the sequence of the analytical chord objects is primarily controlled by the constraints presented above. Only few further constraints are applied on the analytical chord objects: all chords are diatonic chords in C major, the progression starts with the tonic $I$, and it ends in a cadence. Note that in this particular case it so happened that no descending progressions occurred at all. The number of superstrong progressions was explicitly restricted to 20 percent at maximum. The examples section of the Strasheela website (`http://strasheela.sourceforge.net`) contains a page with further results generated by the presented model, which also demonstrate other aspects of Schoenberg's theory. These examples are provided with full source code.

The presented model is highly flexible. It is applicable beyond the common four-voice setting, beyond the conventional triads, and is even suitable for microtonal music. The first author used this model for composing in 31-tone equal temperament, a temperament very close to quarter-comma meantone [5]. Figure 8 shows the beginning of a movement of "Harmony Studies", a 7-limit harmony cadence, which consists solely of ascending chord progressions. Remember that enharmonic spelling indicate different pitches in 31-tone equal temperament. While the interval $C - E\flat$ is the minor third (6/5), the interval $C - D\sharp$ is the subminor third (6/7). In order to assist deciphering the notation, also a harmonic analysis is provided: C harm 7 indicates the harmonic seventh chord over $C$ ($4:5:6:7$, notated in meantone as $C, E, G, A\sharp$), while subharm 6 is a subharmonic sixth chord ($\frac{1}{4} : \frac{1}{5} : \frac{1}{6} : \frac{1}{7}$).

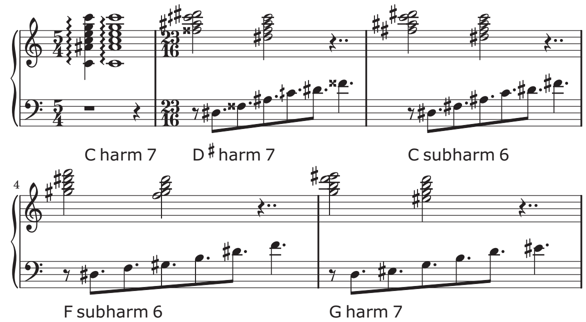


**Figure 8**. Beginning of a movement of "Harmony Studies", notated in 31-tone equal temperament (meantone)

## 5 SUMMARY

This paper detailed a formal model of Schoenberg's root progression guidelines, an important aspect of his Theory of Harmony. Instead of modelling his rules directly (as constraints on scale degree intervals between chord roots), we modelled his explanation of these rules (as constraints between chord pitch class sets and roots).

For chord progressions of diatonic triads in major – the context in which Schoenberg discusses his guidelines – Schoenberg's rules and the proposed model are equivalent. Our constraints can thus be used for implementing exercises proposed by Schoenberg's book as shown above.

However, the behaviour of our model and Schoenberg's rules differ for more complex cases. According to Schoenberg, a progression is superstrong if the root interval proceeds a step up or down. For example, the progression $V^7 - IV$ is superstrong according to Schoenberg. In the presented model, however, this progression is descending! The root of $IV$ is contained in $V^7$ (e.g. in $G^7 - F$, the root pitch class $F$ is already the sevenths of the preceding chord). Indeed, this progression is rare in music. By contrast, the progression $I - III\flat$ (e.g., $C - E\flat$) is a descending progression according to Schoenberg's rules. In our proposed model (the variant $isAscending_2$), this is an ascending progression (the root of $E\flat$ is not contained in $C$), and in our intuitive rating this progression does indeed feel strong.

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