COMPUTER-AIDED TRANSFORMATIONAL ANALYSIS WITH TONE SIEVES

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ABSTRACT

Xenakis' tone sieves belong to the first examples of theoretical tools whose implementational character has contributed to the development of computation in music and musicology. According to Xenakis' original intuition, we distinguish between elementary sieves and compound ones and trace the definition of sieve transformations along the sieve construction. This makes sense if the sieve construction is considered as part of the musical meaning. We explore this by analyzing Scriabin's Study for piano Op. 65 No. 3 by means of intersections and unions of whole-tone and octatonic sieves. On the basis of this example we also demonstrate some aspects of the implementation of sieve-theory in Open-Music and thereby suggest further applications in computer-aided music analysis.

1. INTRODUCTION

According to Iannis Xenakis introductory note to the cello piece *Nomos Alpha* (1966), sieve theory is "a theory which annexes the residual classes and which is derived from an axioms of the universal structure of music". It applies to the formalization of traditional scales as well as microtonal scales, non octaviant scales and any musical phenomenon having a total order structure (intensities, durations, densities, etc.). For exemple, by combining different periodicity by means of classical set-theoretical operations (union, intersection, complementation, symmetric difference), and by interpreting the resulting sieve in the rhythmic domain one can easily "build [...] very complex rhythmic architectures which can even simulate the pseudo-unpredictable distribution of points on a straight line, if the period is long enough"[16]. In fact, as pointed out by the composer in his book *Formalized Music*, "sieve theory is the study of the internal symmetries of a series of points either constructed intuitively, given by observation, or invented completely from moduli of repetition" [17]. Moreover, as the composer already predicted in his thesis defense *Art/Sciences Alloys*, sieve theory is entirely implementable and one of the future research areas will be the computer-aided exploration of the theoretical and analytical aspects of this approach [14]. By analyzing the evolution of computational musicology, starting from André Riotte and Marcel Mesnage computer-aided models of music analysis (see [12] for a collected essay of their theoretical writings), other attemps have been made to apply sieve-theory to other dimensions than pitch [2] and to propose general sieve-theoretical algorithms for the formalization of musical structures (see [16] for some algorithms proposed by Xenakis and [4] for the most recent account of implementational model of sieve-theory). A recent approach to metrical analysis — which is called *inner metric analysis* — is sieve-related as well. It has been proposed by Guerino Mazzola in the context of the software RUBATO [7] and has been further elaborated and discussed in many musical analyses by Anja Volk (Fleischer)[6] and [13]. The building stones of these analyses are local meters, i.e. bounded elementary sieves of onsets within a piece. Section 2 of [8] explores sieve-theoretic aspects of free musical meter (i.e. unbound elementary sieves). The inner metrical analysis is the combinatorial investigation of a complex union of all maximal local meters, i.e. as a compound sieve. Metrical and spectral weights quantify the incidence relation of the bounded or unbounded components, respectively.

In this paper we only focus on the pitch domain and on the computer-aided sieve-theoretical description of chord structures and transformations between them.

2. TONE SIEVES AND THEIR TRANSFORMATIONS

The elementary building stones of Xenakis' *sieves* are discrete affine lines of the kind $a_b = \{ka + b, k \in \mathbb{Z}\}$, i.e. arithmetic sequences of integers. General sieves are built from these elementary ones through the boolean operations of union, intersection and complement. *OpenMusic* visual programming language [1] offers specialized functions and factories to construct sieves and to experiment with them for compositorial or analytical purpose (see section 4).

Our analytical example in section 3 departs from two types of elementary sieves and their complements. On the one hand we consider the *whole-tone* sieves

$$\begin{aligned} 2_0 &= \{..., -4, -2, 0, 2, 4, ...\} \\ 2_1 &= \{..., -3, -1, 1, 3, 5, ...\} \end{aligned} \qquad (1)$$

These are complementary to each other: $2_0^c = 2_1$ and $2_1^c = 2_0$.

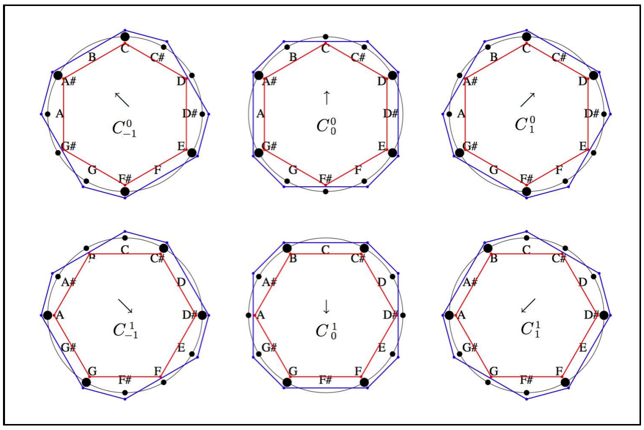

Figure 1. The six configurations of superimposed whole-tone and octatonic sets are graphically represented by configurations of a hexagon and an octagon. To each of them there is an associated intersection sieve $2_m \cap 3^c_n$ (being graphically represented by four thick dots) and a union sieve $2_m \cup 3^c_n$ (being graphically represented by 10 dots).

On the other hand we depart from elementary minor-third sieves

$$\begin{aligned} 3_{-1} &= \{..., -4, -1, 2, 5, 8, ...\} \\ 3_0 &= \{..., -6, -3, 0, 3, 6, ...\} \\ 3_1 &= \{..., -5, -2, 1, 4, 7, ...\} \end{aligned} \tag{2}$$

and consider their *octatonic* complements

$$\begin{aligned} 3^c_{-1} &= \{..., -6, -5, -3, -2, 0, 1, 3, 4...\} \\ 3^c_0 &= \{..., -5, -4, -2, -1, 1, 2, 4, 5, ...\} \\ 3^c_1 &= \{..., -4, -3, -1, 0, 2, 3, 5, 6, ...\} \end{aligned} \tag{3}$$

The term 'octatonic' refers to the period of 12. All boolean constructions with sieves of periods 2 and 3 are in fact 6-periodic, (6 being the least common multiple of 2 and 3), but for reasons of musical analysis we chose the redundant period 12 instead. For any choice of a whole to sieve 2_m and and an octatonic sieve 3^c_n we construct their intersection $2_m \cap 3^c_n$ and union $2_m \cup 3^c_n$ as well. By the symbol C^m_n we denote the 'boolean diamond' of all four sieves with sieve inclusions and call that a *wholetone-octatonic conficuration* (WO-configuration):

$$\begin{tikzcd} & 2_m \cup 3^c_n & \\ 2_m \arrow[ur] & & 3^c_n \arrow[ul] \\ & 2_m \cap 3^c_n \arrow[ul] \arrow[ur] & \end{tikzcd}$$

In total there are six WO-configurations C^m_n because m varies between 0 and 1 and n varies from -1, over 0 to 1. We alternatively use the following arrow-notation for the six WO-configurations. A full overview of all involved sieves for all configurations is displayed in Figure 1.

$$\nwarrow = C^0_{-1} \qquad \uparrow = C^0_0 \qquad \nearrow = C^0_1$$

$$\swarrow = C^1_{-1} \qquad \downarrow = C^1_0 \qquad \searrow = C^1_1$$

bars	1 - 3	4	5 - 8	9 - 11	12	13 - 16
sieves	$\downarrow$	$\swarrow$	$\searrow$	$\swarrow$	$\searrow$	$\downarrow$

Table 1. Sieve-content of bars 1-16

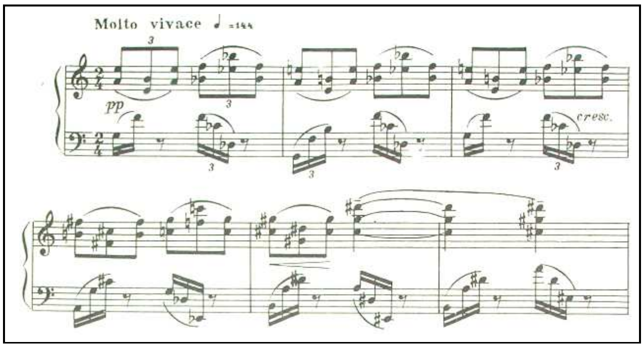

Figure 2. Bars 1- 6 of Scriabin's piano study

This array is useful for the distinction between elementary and compound sieve transformations. Horizontal and vertical connections correspond to the rotation (transposition) of either the octatonic or the whole tone sieves, respectively. Diagonal connections involve a simultaneous rotation of both components. The following analysis of a late piano study of Alexander Scriabin has the interesting property that all successive sieve transformations are elementary.

3. AN ANALYTICAL EXAMPLE

Scriabin's Study for piano Op. 65 No. 3 can be nicely interpretated in terms of the WO-configurations C^m_n and the elementary transformations between them. The association between segments of the piece with these WO-configurations is straight forward and from there the transformational analysis leads to a two-voice "Sieve Counterpoint", where we analyse the piece as a progression of sieve pairs.

3.1. Bars 1 - 16

Bars 1 - 6 exemplify three WO-configurations, namely $\downarrow = C^1_0$ in bar 1 - 3, $\swarrow = C^1_1$ in bar 4 and $\searrow = C^1_{-1}$ in bars 5 - 6 (continuing till bar 8). See Figure 2. The left hand of these segments exemplifies the intersection sieves $2_1 \cap 3^c_0 = \{1, 5, 7, 11\}, 2_1 \cap 3^c_1 = \{3, 5, 9, 11\}$, and $2_1 \cap 3^c_{-1} = \{1, 3, 7, 9\}$. Both hands together exemplify the union sieves $2_1 \cup 3^c_0$, $2_1 \cup 3^c_1$, and $2_1 \cup 3^c_{-1}$ up to two missing tones each. The score in Figure 3 displays a reduction of the bars 1 - 16 , which justifies the WO-configurations in table 1.

3.2. Bars 17 - 62

It follows a passage of 14 bars, which is associated with the opening WO-configuration C^1_0. The A in bar 20 does not belong to the intersection sieve $2_1 \cap 3^c_0$, but it imitates

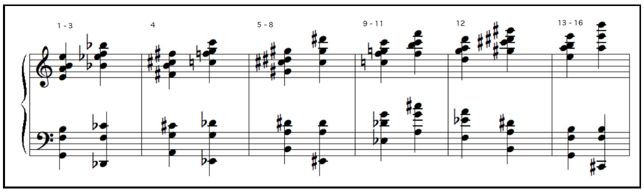

Figure 3. Bars 1- 16 of Scriabin’s piano study

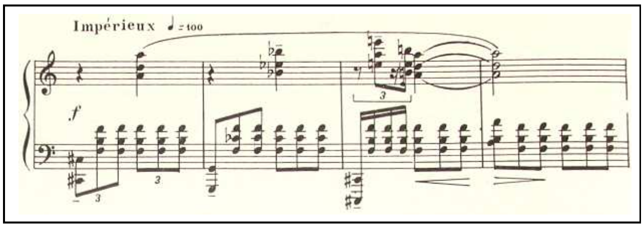

Figure 4. Bars 17 - 20 of Scriabin’s piano study

the A’s in bars 17 and 19 and can therefore be seen as a satellite to the right hand. The tones of both hands together still do not form form the complete sieve $2_1 \cup 3_0^c = \{1, 2, 3, 4, 5, 7, 8, 9, 10, 11\}$. But now only one tone is missing: $8 = A_\flat$. Bars $17 - 20$ illustrate the syntactic situation (see Figure 4). The associated sieves can be verified with the help of the reduction of bars 17 - 62 (see Figure 5).

3.3. Bars 63 - 92 and Coda

Up to a rhythmic detail bars 63 - 92 entirely repeat bars 1 - 30. Thus we have the sieve segmentation in table 2.

The Coda (bars 95 ff.) presents a particularly interesting situation, because of the chromatic run in the right hand, which seems to undermine the fine harmonic structure by a purely melodic mechanics. However, this is not the case. It appears that the trioles in each half bar fit with the left hand chords which themselves descend in minor thirds along the four bars 95 - 98. This results in a corresponding pendulum between the WO-configurations $\downarrow = C_0^1$ and $\uparrow = C_0^0$. Within this process each of the two 10-tone-sieves $2_1 \cup 3_0^c = \{1, 2, 3, 4, 5, 7, 8, 9, 10, 11\}$ and $2_0 \cup 3_0^c = \{0, 1, 2, 4, 5, 6, 7, 8, 10, 11\}$ is fully accumulated.

3.4. A Two-Voice Sieve Counterpoint

Aside from the pure segmentation it is of course interesting to study the transformational behavior of the sieves in their succession. To that end we use the metaphor of a two part counterpoint. Each WO-pair is determined by one out of three states of the octatonic component and by

bars	63-65	66	67-70	71-73	74	75-78	79-92
sieves	↓	↙	↘	↙	↘	↓	↓

Table 2. Sieve segmentation for bars 63-92

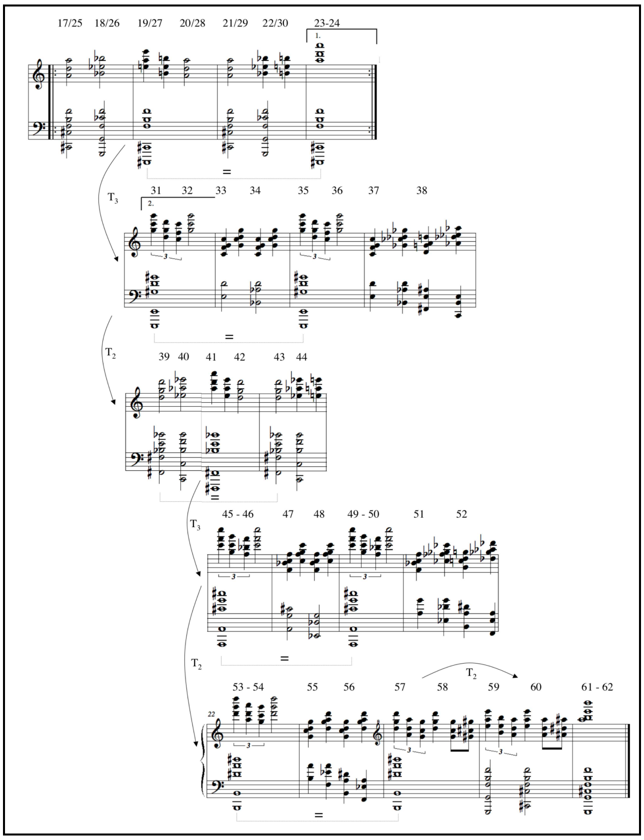

Figure 5. Reduction of bars 17 - 62

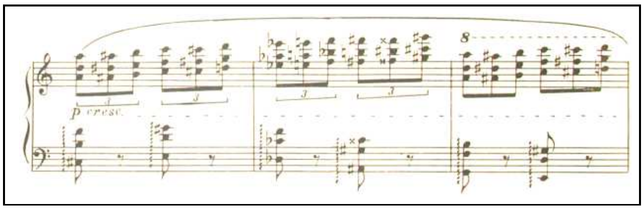

Figure 6. Bars 95 - 97 of Scriabin’s piano study

bars	95	96	97	98	99 - 102
sieves	↓↑	↓↑	↓↑	↓↑	↓

Table 3. Table captions should be placed below the table

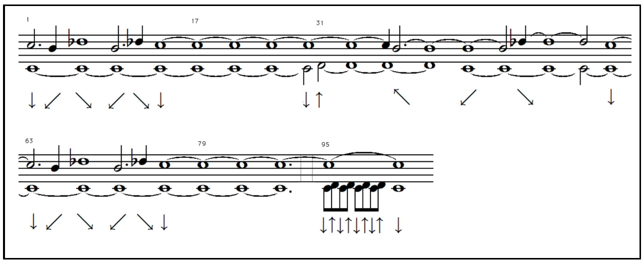

Figure 7. Two-Voice Sieve Counterpoint of the whole piece. The upper voice represents octatonic sieves, the lower voice represents whole tone sieves

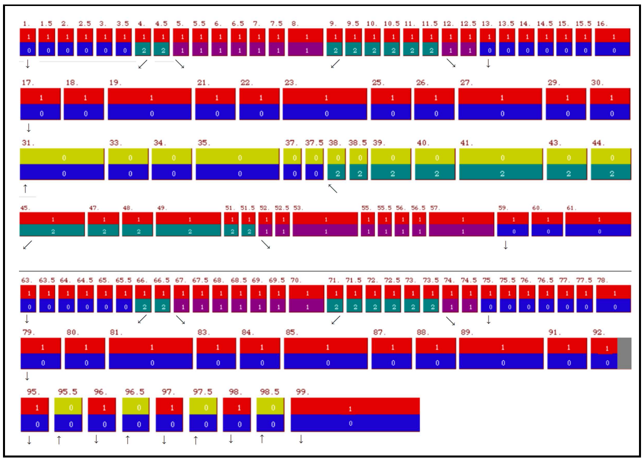

Figure 8. Overview of the analysis of the whole piece

one out of two states of the whole tone component (see Figure7). The following two voice counterpoint encodes the octatonic states in its upper voice (using the tones c_2, b_1, and $d_\flat 2$ for 3_0, 3_1, 3_{-1} respectively) and the whole tone states in its lower voice (using the tones c_1 and d_1 for the 2_1, 2_0 respectively. The WO-configuration "arrow down" = C_0^1 shall be called the central configuration and is meant to be a sieve-theoretic analogue to the traditional concept of "Klangzentrum" in the analysis of Scriabin's late period. In the abstract sieve-counterpoint the concrete tones c_1 and c_2 represent the elementary constituents of the central sieve configuration, i.e. corresponding to the whole tone sieve $\{1, 3, 5, 7, 9, 11\}$ and the octatonic sieve $\{1, 2, 4, 5, 7, 8, 10, 11\}$. In order to avoid confusion between the concrete music and the analytical abstraction we chose tones which are not elements of these sieves. We chose the stable interval of the octave c_1-c_2 in order to express the aspect of centrality, while the other four intervals c_1-b_1, c_1-$d\flat_2$, d_1-c_2, d_1-b_1 represent "out of center"-sieve configurations. The WO-configuration C_1^0 corresponding to the sixth possible interval $d_1 - d\flat_2$ does not occur in the analysis.

As one can immediately observe, all transformations are elementary, i.e. in each succession there is only one voice moving. This indicates the absence of semitone and fifth-transpositions between the sieves throughout the piece.

Cliff Callender [5] argues on the background of investigations into voice leading that the harmonic vocabulary of the late compositions of Alexander Scriabin is located between the wholetone scale and the octatonic. This directly motivates the present study. While Callender considers split voice leadings (single tones going into their chromatic neighbors) connecting octatonic collections with whole tone collections and vice versa. The sieve transformations of the present study connect only sieves of the same kind with one another. Thus it is interesting to compare both approaches in a further study. Another investigation relates the whole tone and octatonic sieves with the left hand chords of Scriabin's piece in terms of topos theory [9], [10] and [11]. In [10] the authors give an informal introduction the the study of transformational logics , where cosieves of transformations represent generalized truth values. The paper Noll [9]presents a more mathematically oriented investigation into this subject and, finally, [11] investiges the links between sieves of tones, such as in this paper, with sieves of triadic transformations.

In this study the piece is divided in small harmonic segments such as half bars and sometimes larger segments (as in bar 17 and following bars). To each harmonic segment one may attribute exactly one pair of indices m and n such that the left hand tones are contained in the corresponding intersection sieve $2_m \cap 3_n^c$ and that the tones of both hands together are contained in the corresponding union sieve $2_m \cup 3_n^c$. Figure 8 displays the global harmonic organization of the whole piece. For each harmonic segment there is exactly one pair of indices n and m, such that the union sieve $2_m \cup 3_n^c$ covers the all pitch classes of both hands and the intersection sieve $2_m \cap 3_n^c$ covers the left hand pitch classes of the corresponding segment.

This segmentation is a proper refinement to the segmentation into maximal sieve-extensions. The former one has less dense segments within the sieves but it is more sound with the topos-theoretic considerations of [9] as well as with the voice leading considerations of [5]. [1]

4. SIEVE CONSTRUCTIONS IN *OPENMUSIC*

We now present some aspects of a recent implementation of sieve-theoretical models in *OpenMusic* visual programming language [1]. This environment for computer-aided music theory, analysis and composition has been integrated as a package of mathematical tools (MathTools) in the last version 5.0 of *OpenMusic*. In a more general way, the MathTools environment enables the construction of algebraic models of music-theoretical, analytical and compositional processes. Its "paradigmatic" architecture, taking several different group actions as the basis of variable catalogues of musical structures, enables to give a formalized and flexible description of the notion of "musical equivalence".

This makes use of some standard algebraic structures

[1] The *OpenMusic* presentation includes a maquette, where each small or large segment can be played and interactively investigated.

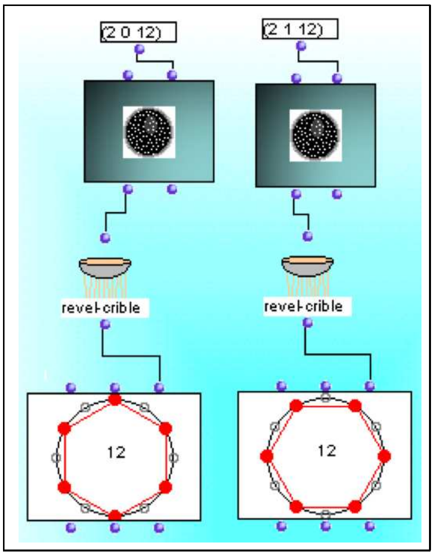

Figure 9. OpenMusic implementation of the complementary whole-tone sieves.

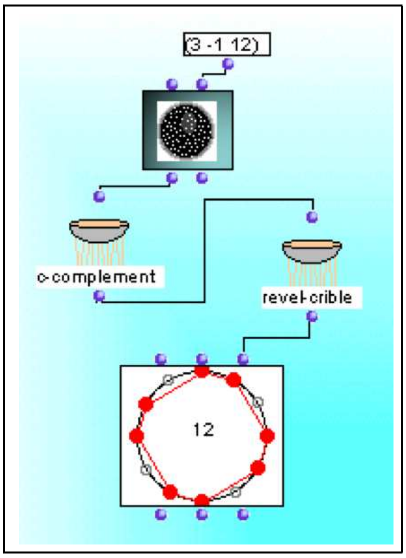

Figure 10. OpenMusic implementation of the octatonic scale.

(cyclic, dihedral, affine and symmetric groups) as well as more complex constructions based on the ring structure of polynomials. In this package, there are six main families of functions, which are: circle, sieves, groups, sequences, polynomials, canons. In a previous paper [3] we focused on four families of tools which were strictly connected with the problem of paradigmatic classification of musical structures (the circular representation, groups and polynomials).

Although from a mathematical point of view sieves are infinite ordered structures, the sieve theoretical construction we used for the analysis of Scriabin's *Study* Op. 65 No. 3 are isomorphic to subsets of the finite cyclic group of order 12. For this reason, we can easily represent the sieves by means of the circular representation. Figure 9 shows the *OpenMusic* implementation of the complementary whole-tone sieves of equation (1).

Figure 10 shows the constructions and musical representation of the first octatonic sieve in equation (3) starting from its minor-third complements. Notice that the same octatonic sieve could be constructed as the set-theoretical union of two minor-thirds sieves (Figure 11).

By using set-theoretical intersections and unions, we can graphically represented the process leading, for exemple, to the construction of $2_0 \cap 3_0^c$ and $2_0 \cup 3_0^c$ (Figure 12).

Starting from the circular representation, sieves can also be represented in traditional musical notation via the function `c2chord` which maps the geometric representation of a given chord into a chord or a rhythmic pattern. Figure 13 shows the pitch and rhythmic representation of the sieve $2_0 \cup 3_0^c$.

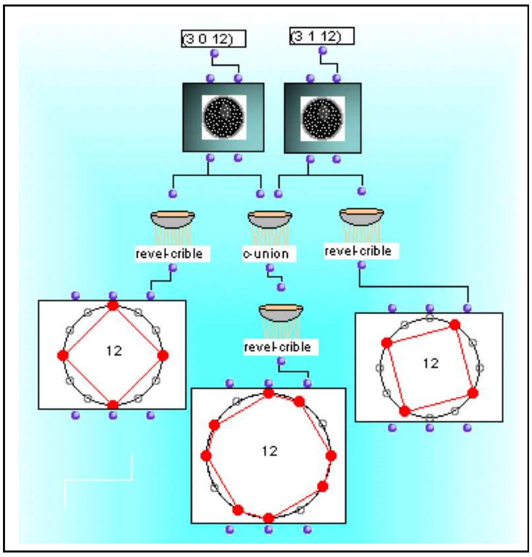

Figure 11. A different set-theoretical construction of the octatonic scale.

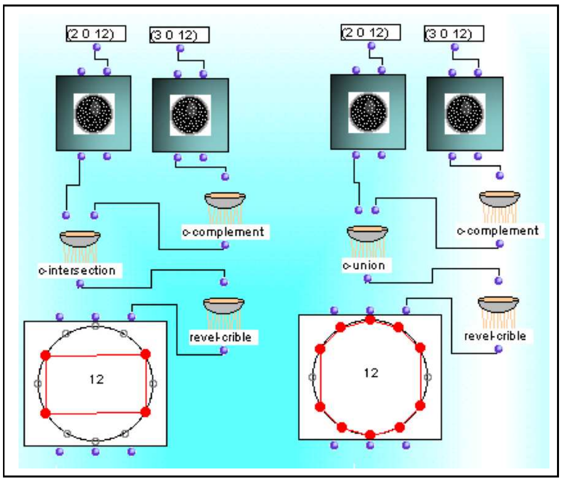

Figure 12. OpenMusic implementation of the sieves $2_0 \cap 3_0^c$ and $2_0 \cup 3_0^c$.

5. CONCLUSIONS

It is very likely that the sieve analysis of the chosen example by Alexander Scriabin does not represent a poi-

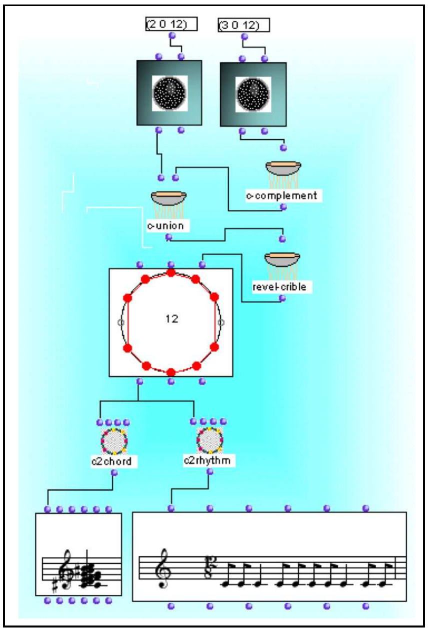

Figure 13. Pitch and rhythmic representation of the sieve $2_0 \cup 3_0^c$.

etic perspective. But on a neutral level of analysis it is quite convincing and pedagogically more convincing than some of Xenakis own examples. Furthermore it suggests a more systematic study of partial transformations in complex sieve constructions, i.e. the independent transformations of elementary components of compound sieves. In our examples the partial transformations represent a special case of transpositions, but generally this will not be the case: a transposition of a defining component of a compound sieve does not necessarily result in a transposition of the compound sieve.

Sieve-theoretical models have both a pedagogical and a musicological interest for they enable the music theorist to visualize some structural musical properties in a geometric way and to test the relevance of different segmentations in music analysis. This could have a strong implication in the way to teach music theory, analysis and composition.

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