CONSTRAINS ON THE MODEL FOR SELF-SUSTAINED SOUNDS GENERATED BY ORGAN PIPE INFERRED BY INDEPENDENT COMPONENT ANALYSIS

E. De Lauro *S. De Martino* *M. Falanga* *G. Sarno*

Department of Physics, Salerno University, S. Allende, I-84081, Baronissi (SA) ITALY

ABSTRACT

We use Independent Component Analysis, an entropy based method to study self-sustained sound generated by organ pipe and recorded in an anechoic chamber. We recover a simple analogical model that can be ascribed to the same types of Woodhouse, Yoshikawa models [3,11]. Our model is able to reproduce in the listening the generated sound. Then we use self-sustained oscillators as a source for linear organ pipe models and we observe that the real signals can be reproduced only considering nonlinear interaction between the different constituents of an organ pipe.

1. INTRODUCTION

Musical instruments and their sounds mechanism production have attracted the attention of scientists since very long time [1,2]. The musical instruments are innumerable, constructed by all people in every time. Despite this variety, in a first approach and looking at their general features, they can be divided into two large classes. Those producing sounds which decay freely and those able to produce sustained tones. To the first class, we ascribe e.g. piano or guitar; violin, clarinet or organ pipe e.g. are in the second class.
The first attempt to produce theoretical models for sounds production, as it is natural, relies upon linear harmonic approximation. The general features of first class are well reproduced within this linear approach. But it fails when it is applied to second class, i.e. sustained-tone instruments. The organ pipe, among sustained-tone instruments, is one of the most studied. It can be considered representative of a large class of flue instruments. But, we do not have to consider the nonlinear instrument–players interaction, that can be very significant in producing tone, as in the case of other instruments as clarinet and so on. The linear analysis techniques and modelling, as modal analysis, finite element analysis, wave propagation and dispersion, reflection and transmission at impedance discontinuities, have given significant results. The commonly accepted model for flue instruments is jet-drive model [12-13]. It is based on the dynamics of air jet interacting with sharp edge placed in the mouth of resonator. This model has been implemented, along the time, to reproduce many of the characteristics of produced sounds looking at their Fourier transform or to the observable jet amplitude.
It appears to explain for example the link between the blown frequency and the organ pipes "modes" or the tendency to excite higher "modes" increasing driving pressure.
In this approach, the body of pipe is considered as a passive, linear "resonator". Its mechanical vibration does not play any significant role in the sound generation mechanism.
This approach, in order to explain the slight differences between harmonic series frequency and pipe "modes", introduces end-corrections with frequency.
It is customary that the self sustained instruments could not function without intrinsic nonlinearity in their underlying behaviour. To compare theoretical model with Fourier transform of true signals can be misleading.
It is well known that it is possible to produce nonlinear signals with very different waveforms but characterised by the same frequency content.
In this view, one needs an approach able to reproduce the recorded sound waveforms in the time. As a first step, we think that the analysis of sustained tone instruments must be supplemented by some nonlinear techniques in time domain, to state what are the true nonlinear characteristics of acoustic field.
In this paper we apply a method of neural networks analysis to sound radiation of organ pipe, namely Independent Component Analysis (ICA) (for more details see [4] and many papers therein cited). This tool works in the time domain to analyse linear mixture of even nonlinear signals. We make an experiment considering different conditions as explained in the next section. Then we use ICA to study the experimental recorded signals.
This is the second experiment that we make. The results of the first experiment are reported in [5].
As there is a great variety of organ pipes, we consider in particular flue pipes. Moreover, flue pipes may be open, stopped or partially stopped. They are made of wood or metal and so on. We restrict our analysis to one of the more diffused type i.e. open metal flue pipe and we consider, in particular, notes produced in the C rank.

2. DATA SET AND LINEAR SPECTRAL ANALYSIS

We have recorded in an anechoic chamber different notes by using a digital multi-track with the sampling frequency of 44100 Hz and 6 microphones.

We focus our attention on D note, to study what is changing when different pressure conditions are involved. The experiment has been performed with the aim to excite higher and higher modes as pressure increases.

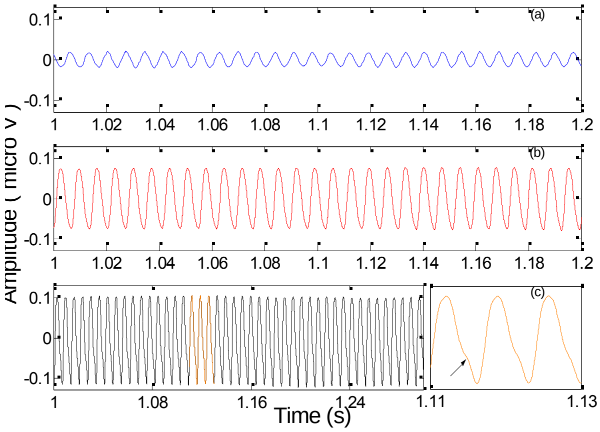

Figure 1. Time evolution of recorded D note under different increasing values of pressure: (a) lowest (blue line), (b) intermediate (red line) and (c) pressure corresponding to the full played note (black line). We underline the distortion of the waveform increasing pressure as one can see in the zoom on the right (orange line).

In Fig. 1, three examples of different increasing values of pressure (low, intermediate, and full driven pressure) are represented both in time and frequency domain.

In details, in Fig. 1a-1c, we report an example of the D note recorded at the three different values of pressure. As we can see the time signals increase in amplitude but are also with different waveforms.

If we look at spectra (Fig. 2), we observe in all three cases (broadband spectrum with many peaks), but with different amplitudes on the first three peaks. It is possible also to see in Fig. 2 the shift of the fundamental harmonic toward high frequencies when pressure increases. All this is well known, if we consider spectra. Our aim is to analyse the three different time signals in time domain by using ICA.

3. NONLINEAR ANALYSIS: INDEPENDENT COMPONENT ANALYSIS

Before to apply we give some details about ICA technique. ICA is a method to find underlying factors or components from multivariate (multidimensional) statistical data [4]. It is a well defined method in speech context, to solve the classical problem of cocktail party. Imagine to have some people speaking in a room and some microphones recording their voices. The goal of ICA is to extract, from the mixtures of voices, step by step, each independent speaker.

Let us explain in brief the mathematical setting on which ICA is based. We can suppose to have m different recorded time series $\mathbf{x}$ (our recordings), that we hypothesize to be the linear superposition of n mutually independent unknown sources $\mathbf{s}$ (independent "modes"), due to different mixing, represented by a constant unknown $m \times n$ matrix $\mathbf{A}$.

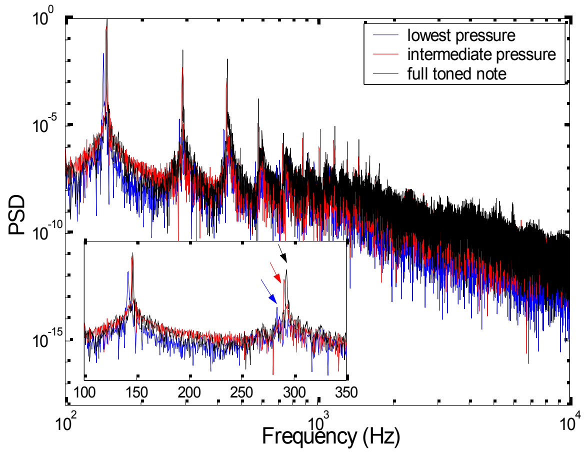

Figure 2. The normalized spectra are reported in order to compare the different situations. The appearance of higher peaks and the trend towards the high frequencies is shown by the arrows in the inset that is a zoom.

The mixing is essentially due to the medium between the sound source (i.e. organ pipe) and the microphones and to some background noise. The hypothesis is to have instantaneous linear mixtures of some independent source signals. To satisfy the request of instantaneous recordings at our 6 microphones, we correct for the time propagation of sound in the air. If the mixing has to be linear, nothing is assumed with respect to the sources signals, which can be linear or nonlinear or eventually noise. We remark that amplitudes are not preserved. In addition to the source independence request, ICA assumes that the number of available different mixtures m is at least as large as the number of sources n. The ICA is based on the central limit theorem: the distribution of a sum of independent random variables tends towards a Gaussian distribution, under certain conditions. Hence, the separation is achieved maximizing the non-Gaussianity (super-Gaussianity or sub-Gaussianity) of the mixtures, so that independent components can be extracted. On the other words, maximizing the non-Gaussianity allows to find one independent signal. The statistical independence of the components is evaluated by using fourth-order statistical properties. We apply the *fixed-point* algorithm, namely FastICA [6].

We have used the basic model of ICA. It is possible, however, to take into account different models: the number of mixtures is less than the number of sources [7]; convolved signals (signals with delays and echoes); nonlinear mixing [8].

For their stationarity, self-sustained tone acoustic signals are suitable to be analysed by using ICA. Hence, we can apply ICA to understand if a full toned note can be decomposed into independent separated signals. As a consequence, we have the possibility to investigate the kind of coupling among them.

Applying ICA to the three considered cases i.e. low, intermediate and full driven pressure, we obtain the following results:

1) in the first case (low pressure), we have one extracted component in time domain corresponding to the fundamental at 142 Hz (Fig. 3);

2) in the second case (intermediate pressure), two independent modes are extracted, corresponding to the principal at 145 Hz and the first higher “harmonic” at 290 Hz (Fig. 4);
3) in the third (full driven pressure), only three among all the peaks present in the spectrum (Fig. 5) are separated: the fundamental (146 Hz), the first (292 Hz) and the second harmonic (439 Hz).

We would like to underline this last case. It confirms the result of our first experiment: this kind of pipe produces, when it is played in a full tone, three “modes” in the form of three independent limit cycles in time domain. This suggests that these nonlinear “modes” (limit cycles) are activated separately, and weakly coupled in the full toned recorded note.

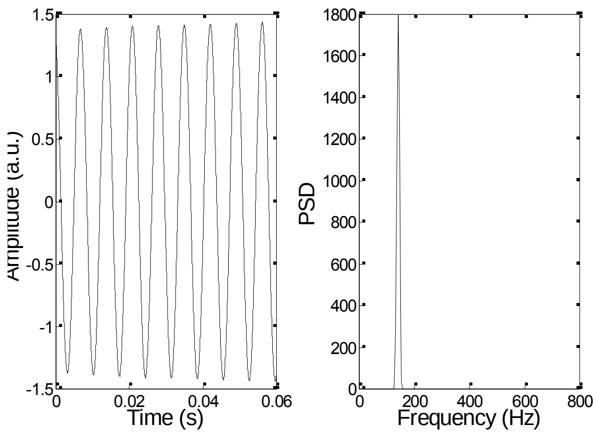

Figure 3. The only extracted component corresponds to the fundamental mode (at 147 Hz); on the right its spectrum is plotted. Note that the spectrum is on linear scale to better evidence the predominance of this frequency with respect to the others, which appear in loglog scale due the nonlinear waveform (limit cycle).

In the results presented in [5], we have considered the comparison for E note produced by open pipe and recorded both in a church and in an anechoic chamber.

Then the higher “Landau” modes cannot be considered as due to resonance associated to the environment. They are effectively three self-oscillations produced by a system in a regime of weak coupling.
Summing up we have made an experiment changing the external air pumping level; we start with zero pressure at the blowing apparatus, then we increase until we generate sound. Then we increase when sound changes perceptively, then we arrive to the maximum blowing as suggested by organ builders.
We obtain the three observable states that we have analysed with ICA (see items 1,2,3).
It is very interesting to note that in the anechoic chamber where there is not environment disturbing, people perceive exactly the time in which start the higher Landau modes.
We remember that, in the standard theory of sound production, to explain the distortion of waveform, a strong coupling among all modes has been supposed [2].
This is completely in disagreement with our experiment, namely our experiment prove that there are as components of full toned note three weakly coupled self-sustained oscillations.

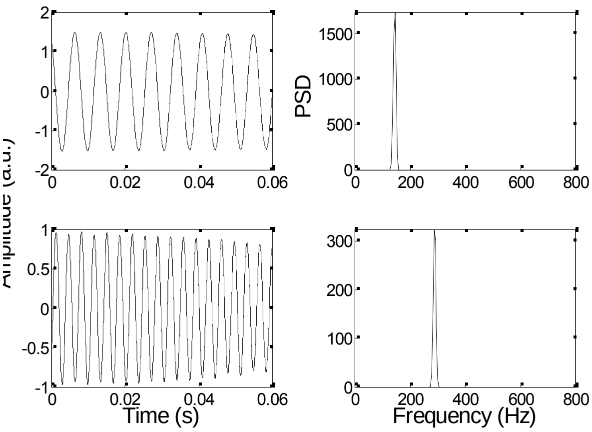

Figure 4. Extracted components and their spectra. The two extracted components correspond respectively to the fundamental mode (at 145 Hz) and to the first higher mode (at 290 Hz).

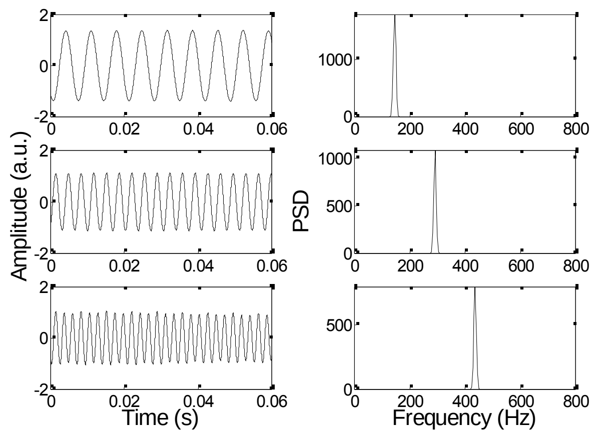

Figure 5. Extracted components and their spectra. The components correspond respectively to the fundamental mode (at 146 Hz) and to the higher modes (at 290 Hz and 439 Hz).

4. SINGLE MODE MODEL

In this section, we want to prove that effectively we are speaking of self-sustained oscillations. Then, our first aim is to obtain simulated signals that can reproduce in the listening the recorded notes. We know on the basis of ICA analysis that a full toned note consists of three linearly weak coupled self-sustained oscillations. In other words, we know that the sounds, produced by organ pipe, are settled by means of nonlinear mechanism, in the limit cycle regime. We can consider a particular simple oscillator producing self-sustained oscillations, i.e. the Andronov oscillator [9]. This oscillator generates, with suitable parameters, a limit cycle, which is approached asymptotically by all other phase paths. This limit cycle is dynamically stable. It is representative of many nonlinear systems with feedback.
The equations of the Andronov oscillator with natural angular frequency ω_O, in the suitable form, are:

$$\begin{aligned}\ddot{x}+2h_1\dot{x}+\omega_o^2 x=0 \quad x<b\\ \ddot{x}-2h_2\dot{x}+\omega_o^2 x=0 \quad x>b,\end{aligned} \tag{1}$$

where x is representative of the pressure of air in the case of pipe or clarinet, or of the string velocity in the case of bowed string instruments like violin; ω_0 is the fundamental mode. The nonlinearity is contained in a suitable threshold b ruling the self-coupling. A discussion of the physical meaning of h_1 and h_2 is out of our purpose, but it is clear that these two parameters represent, in an effective way, pumping due to the pressure entering in the pipe and all the typical dissipating effects always present in the organ pipe sound production.
Generally speaking, Andronov oscillator has different behaviours as varying parameters and threshold. In fact, if the threshold is negative, we can have a limit cycle or forcing oscillations, while, if the threshold is positive, no limit cycle is obtained.
We remind that the general solution of this oscillator can be cast in the following form, i.e. in the general form proposed by Woodhouse for all self-sustained sounds:

$$x(t)=\int G(t-\tau)f(x(\tau))d\tau, \tag{2}$$

G(t-τ) is a suitable propagator and f(τ) represents the self-coupling (e.g. the friction low depending on velocity if we are dealing with violin, or the interaction between flux and pressure in the case of pipe).
Setting the natural frequency to the value corresponding to the fundamental in the recorded D note, we have to fix the parameters b, h_1 and h_2 in order to have a very high superposition degree with the played one mode signal. To estimate the parameters h_1 and h_2 of the single Andronov oscillator, we construct a tri-dimensional matrix, whose elements generate, separately, a signal that can be compared to the original one mode signal. We choose the best tern in a sense of minimum square, i.e. we fix those parameters that generate a minimum root mean square deviation with respect to the reference signal. The values of the parameters are the following: b=-0.017; h_1=1990, h_2=100 for the fundamental mode, correlation coefficient is 0.86. The result is reported in Fig. 6. Now we want to show as can be misleading. Starting from the value of previous parameters, we increase the driving pressure, that here represented by the threshold b. We control our theoretical system looking at Fourier transform until we arrive to two FT for true full toned note and that relative to our simulating model that are completely superposed (see Fig. 7).
Then, we should conclude that the true signal relative to the full toned note is well reproduced by our analogical model. But if we apply ICA to different mixtures obtained little changing the h_1, h_2 and b parameters of Eq. 1, we do not reach any separation among the different modes, even if the FFT is well reproduced. This means that, by itself, this simple system is not enough to reproduce the real signals.
It needs a linear superposition of three Andronov oscillators to reproduce the true full toned note.

This means that analogical system representing the real signal is a tri-dimensional system made of three linearly low coupled nonlinear oscillators. The parameters h_1, h_2 and b are the same, namely they are linked to general organ pipe structure, the frequency changes.
In Fig. 8 we show this superposition, making a comparison with the played note. As we can see the correlation coefficient is very high.

5. CONCLUSION

On the basis of our experiment we can conclude that, in first approximation, the low dimensional dynamical system representing on average the fluid-dynamical equations modelling organ pipe is constituted by three linearly weakly coupled Andronov oscillators. It is very simple to recover a general model starting from our experimental results. Namely, we can write wave equations with a dissipative term and nonlinear source, made by a linear combination with suitable coefficient of three Andronov oscillators corresponding to the three Landau modes. This model obviously reproduces the sounds namely this equation is constructed such a way to relax on source. We effectively have constructed this simple model comparing the observed signals with produced signals. But this model cannot explain the appearance of higher harmonics with the growth of external pressure. Then the true model must generate the three modes through an intrinsic nonlinear coupling between the activation apparatus (labium, edge, languid) and the organ pipe body. We think that the elastic vibration of organ pipe body cannot be irrelevant in the generation of tone. Namely, we have made another experiment that we report in a separate paper. In this experiment if we superimpose our hand on the organ pipe body, we modify drastically the time waveform.

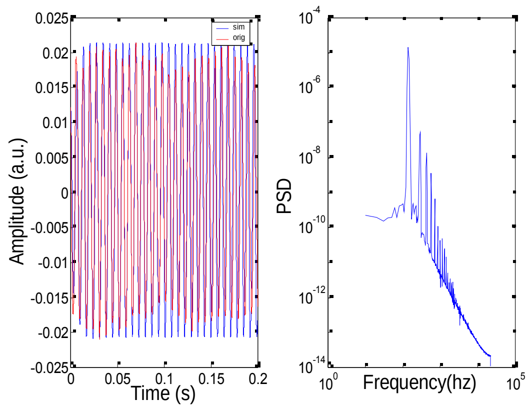

Figure 6. Superposition of *one* mode note (red line) and simulated note (blu line). The correlation is 0.86.

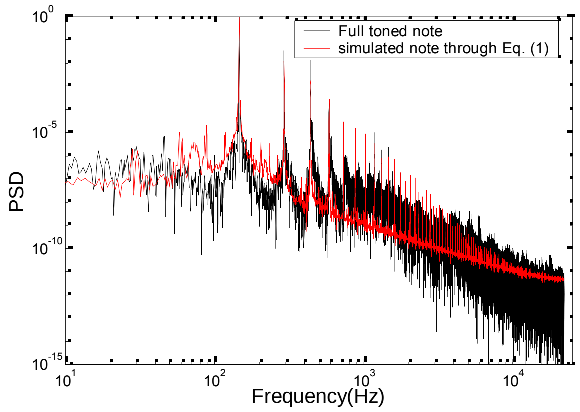

Figure 7. Superposition of full toned note (black line) and simulated note (red line). The main features are well represented.

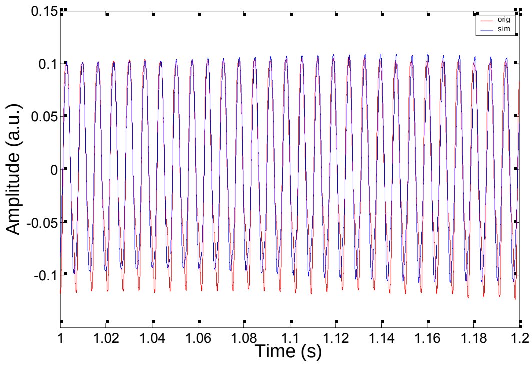

Figure 8. Comparison between the played note (red line) and the superposition of three Andronov oscillators, each corresponding to a single "mode" (blue line). The correlation coefficient is very high (0.95).

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